Plural .	First Person . .	Where go we, incl.	Ngoondeeneeñnun
		Where go we, excl.	Ngoondeeneeñulla
	Second Person	. Where go ye	Ngoondeeneeñoo
	Third Person .	. Where go they	Ngoondeeneeyoolung

Adverbial meanings are sometimes conveyed by means of verbs, as beetyballeemañ, he (or it) goes out of sight. Conjunctions and interjections are few and unimportant.

NOTES ON PURE CIRCULATING DECIMALS.

BY C. A. M. FENNELL, CAMBRIDGE, ENGLAND.

(*Read October 4, 1901.*)

§ 1. The following properties of cyclic periods of decimals are supplementary to those discussed by Prof. Glaisher in the *Proceedings* of the Cambridge Philosophical Society, October 28, 1878, Vol. III, Part v.

§ 2. The following letters, definitions and theorem are taken from p. 185 of Prof. Glaisher's paper. The periods that arise from the series of fractions $\frac{p}{q}$, $\frac{p}{q}$ being a vulgar fraction in its lowest terms, and p having all values less than q (which is prime to 10), are called the periods of the denominator q, or, more simply, the periods of q. Theorem: the denominator $\varphi(q)$, which includes all the above values of p, has a certain number (n) of periods, each containing the same number (a) of digits, n and a being connected by the relation, $na = \varphi(q)$.

§ 3. (i) The first inquiry relates to the distribution of the several digits, 0, 9, 3, 6, 1, 8, 2, 7, 4, 5, over the n periods of a digits which constitute Prof. Glaisher's $\varphi(q)$. In this particular a difference emerges between 0, 9, 3, 6, and the rest of the digits, the observation of which may prove important to the theory of numbers.

Of course there must always be as many 9s as 0s, 3s as 6s, 1s as 8s, etc., but as verified up to $\frac{1}{101}$ there are the same number, say m, of each of the six digits, 1, 8, 2, 7, 4, 5, m being a positive integer.

E.g., in the single period of $\frac{1}{7}$, viz., $.\dot{1}4285\dot{7}$, each of the six occurs once, and the same with the six periods of $\frac{1}{7}$, viz., $.\dot{1}$, $.\dot{2}$, $.\dot{4}$, $.\dot{5}$, $.\dot{7}$, $.\dot{8}$. In the 5 periods of $\frac{1}{11}$, viz., $.\dot{0}\dot{9}$, $.\dot{1}\dot{8}$, $.\dot{2}\dot{7}$, $.\dot{3}\dot{6}$, $.\dot{4}\dot{5}$, every digit occurs once.

(ii) As might be expected, when *an* or $\varphi(q)$ is an exact multiple of 10, each of the 10 digits occurs an equal number of times. In other words, if *an* or $\varphi(q) = 10m$, then each digit occurs m times.

(iii) Prof. Glaisher writes: "Among . . . . results which are illustrated by Mr. Goodwyn's tables . . . . of less importance may be noticed the following: If q be a prime ending with one, viz., $= 10m + 1$, then each of the digits 0, 1, 2 . . . . 9 occurs m times in the $10m$ digits which form the periods of q." This is a partial statement included under the statement in my immediately preceding paragraph and only embracing the cases in which *an* or $\varphi(q) = q - 1 = 10m$.

It seems a safe inference that my more general statement and its place in the methodical distribution of digits in the periods of q, which is based on the forms both of q and of *an* or $\varphi(q)$, were not known when Prof. Glaisher wrote as above, and I have reason for believing that they have not been discovered since, or at any rate published since.

(iv) The said methodical distribution of the several digits, so far as traced at present, comprises at least seventeen distinct divisions of cases which fall into five groups, A, B, ... E.

The results have been verified for all values of q from 3 to 401 inclusive, and for sundry higher values, *e.g.*, 419, 423, 487, 507, 603 and 621.

(v)

A 1. If *an* or $\varphi(q) = 10m$, then for all values of q each of the digits 0, 1, 2, ... 9 occurs m times in the period or periods of q.

B { 2. If *an* or $\varphi(q) = 10m + 2$ and $q =$ either $10\beta + 1$ or $10\beta + 7$, then 0, 9 occur $m + 1$ times each, and the other digits m times each. [But for $q = 357$ (*an* $= 192$), 0, 9, 3, 6 occur 18 times ($m - 1$) each, and the other digits 20 times ($m + 1$)].

B

3. If an or $\varphi(q) = 10m + 2$ and $q = 10\beta + 3$ or $10\beta + 9$, then 3, 6 occur $m + 1$ times and the other digits m times each.

C

4. If an or $\varphi(q) = 10m - 4$ and $q = 10\beta + 1$, then 0, 9, 3, 6 occur $m + 1$ times each and the other digits m times each.

5. If an or $\varphi(q) = 10m + 4$ and $q = 10\beta + 3$, then 0, 9, 3, 6 occur $m + 1$ times each, and the other six digits m times each.

6. If, however, q is a multiple of 3, then 3, 6 occur $m + 2$ times each, and the other digits m times each.

7. If an or $\varphi(q) = 10m + 4$ and $10\beta + 7$ or $10\beta + 9$, then 0, 9 occur m times each, 3, 6 $m - 1$ times each and the other digits $m + 1$ times each.

D

8. If an or $\varphi(q) = 10m + 6$ and $q = 10\beta + 1$ or $10\beta + 3$, or $10\beta + 9$, then 0, 9 occur $m + 1$ times each, 3, 6 $m + 2$ times each and the other digits m times each.

9. If an or $\varphi(q) = 10m - 6$ and $q = 10\beta + 7$, then 0, 9, 3, 6 occur m times each, and the other digits $m + 1$ times each.

10. But if $q = 10\beta + 7 = 3\delta$ or $(10^\theta + 3)\delta$, then 3, 6 occur $m - 1$ times each, and the other digits $m + 1$ times each.

11. If an or $\varphi(q) = 10m - 6$ and $q = 10\beta + 9 = (10\gamma + 3)^2$, then 0, 9, 3, 6 occur m times each, and the other digits $m + 1$ times each.

D

12. But in other cases either 0, 9 occur $m + 1$ times each, 3, 6 $m + 2$ times each, and the other digits m times each;

13. or 0, 9 occur $m - 1$ times each, and the other digits $m + 1$ times each.

E

14. If an or $\varphi(q) = 10m + 8$ and $q = 10\beta + 1$ or $10\beta + 3$, then 0, 9, 3, 6 occur $m + 2$ times each, and the other digits m times each.

15. If an or $\varphi(q) = 10m + 8$ and $q = 10\beta + 7$, then 3, 6 occur m times each, and the other digits $m + 1$ times each.

16. If an or $\varphi(q) = 10m + 8$ and $q = 10\beta + 9$, then 0, 9 occur m times each, and the other digits $m + 1$ times each.

17. But if $q = 10\beta + 9 = 11\delta + 1$ (*e.g.*, 89, 199 or 419), then 3, 6 occur $m - 4$ times, and the other digits $m + 2$ times each, or some other exceptional distribution is found.

(vi) The total number of values of q up to 401 is 160.

A 1. Includes 18 primes (counting 401) and 22 multiples or powers of primes.

B 2. No primes; 15 cases with $q = 10\beta + 1$, only 3 cases with $q = 10\beta + 7$, and the exceptional case $q = 357 = 3 \times 7 \times 17$. Beyond 401, $q = 507$ is regular. But the limits of the investigation do not present sufficient data for sound inference as to the cases where $q = 10\beta + 7$.

B 3. Includes $q = 243$ and 20 primes with 3 for the unit digit and 2 with 9 for the unit digit, namely $49 = 7^2$ and $289 = 17^2$, the next number being $819 = 7 \times 9 \times 13$.

C 4. One case, $q = 81$.

C 5. One case, $q = 343$.

C 6. One case, $q = 273$.

C 7. Five cases, $q = 147$, and 4 cases, $q = 10\beta + 9$.

D 8. Nine cases.

D 9. Twenty-three cases.

D 10. Eight cases, 57, 87, 177, 237, 247, 267, 327, 387.

D 11. Two cases, $q = 3^2$ and $q = 13^2$.

D 12. Three cases, $q = 119$ and $q = 259$, $q = 329$.

D 13. One case, $q = 399$.

E 14. Five cases, $q = 273$, $q = 343$, $q = 133$, $q = 203$, $q = 353$.

E 15. One case, $q = 27$.

E 16. Sixteen cases.

E 17. Two cases, $q = 89$, $q = 199$.

There is then a strong *prima facie* case in favor of a regular classification of the numerical distribution of the digits in various cases of $\varphi(q)$, but not a sufficient number of cases at present investigated for a complete and certain induction, which would moreover demand an explanation of the causes which lead to the observed results. A complete investigation would probably supply eight or ten more divisions of cases, as C 4, C 5, C 6, D 13, E 15, E 17 are probably susceptible of subdivision, and under B 2 the case $q = 357$ may be the lowest case of a distinct division.

The possibility of occasional exceptions must be frankly admitted, at any rate for the present.

EXAMPLES.

(vii) For $q = 3^4 = 81$, $\varphi(q) = 54$, $a = 9$, $n = 6$, the periods are

$.\dot{0}1234567\dot{9}$ containing all the digits except 8.
$.\dot{9}8765432\dot{0}$ " " " " 1.
$.\dot{0}2469135\dot{8}$ " " " " 7.
$.\dot{9}7530864\dot{1}$ " " " " 2.
$.\dot{0}4938271\dot{6}$ " " " " 5.
$.\dot{9}5061728\dot{3}$ " " " " 4.

Therefore obviously 0, 9, 3, 6 occur 6 times each and the other six digits 5 times each.

For $q = 3 \times 11 = 33$, $\varphi(q) = 20$, $a = 2$, $n = 10$, the periods are $.\dot{0}\dot{3}$, $.\dot{0}\dot{6}$, $.\dot{1}\dot{2}$, $.\dot{1}\dot{5}$, $.\dot{2}\dot{1}$, $.\dot{3}\dot{9}$, $.\dot{4}\dot{8}$, $.\dot{5}\dot{7}$, $.\dot{6}\dot{9}$, $.\dot{7}\dot{8}$, in which every digit occurs twice.

For $q = 31$, $\varphi(q) = q - 1 = 30$, $a = 15$, $n = 2$, the periods are $.\dot{0}3225806451612\dot{9}$ and

$.96774193548387\dot{0}$, in which every digit occurs 3 times, each pair of complements of 9 contributing 3 digits to each period.

§ 4. The phenomena noted and illustrated in the following paragraphs can be doubtless fully classified and explained by specialists in the theory of numbers:

(i) If when q is prime its period is divisible into sections, each of which contains an equal number of digits—the number being greater than 1—the sum of the sections arranged in column amounts to $10^d - 1$ or a multiple of $10^d - 1$, where d is the number of digits in each section, and the sum of the numerators corresponding to the periods which begin with the several sections is q or a multiple of q.

E.g., for the period of 31, $a = 15$ and $n = 2$, and written in column of 5 sections of 3 digits each the period of $\frac{1}{31}$ is

$$\begin{array}{l} .\dot{0}32 \\ 258 \\ 064 \\ 516 \\ 12\dot{9} = 999; \end{array}$$

and in column of 3 sections of 5 digits each is

$$\begin{array}{l} .\dot{0}3225 \\ 80645 \\ 1612\dot{9} = 99999; \end{array}$$

while the five enumerators answering to the sections of 3 digits are 1, 8, 2, 16, $4 = 31$, and those answering to the sections of 5 digits are 1, 25, $5 = 31$.

For the period of 7, which is $.\dot{1}4285\dot{7}$ (for which figures see § 6), $14 + 28 + 57 = 99$; $85 + 71 + 42 = 198$; while $142 + 857 = 999$. In the latter case the first half and the second half of the period are complementary. This is an instance of the simplest and most obvious case of the sum of sections of a period being $= 10^d - 1$, and this case must occur, whether q be prime or not, whenever a complementary remainder occurs in the division of p by q. This particular case of complementary halves of a period is not brought under a general theorem relating to sections of periods by Prof. Glaisher.

This property of sections of a period containing an equal number of digits each depends upon the property of the corresponding numerators, viz., that their sum is equal to q or a multiple of q;

for it is obvious that the sum of the sections is equal to the sum of the whole periods which begin with the respective sections.

E.g., for the period of $\frac{3}{31}$

$$\begin{array}{rrrrrl}
.\dot{0}96 & 774 & 193 & 548 & 38\dot{7} & \\
.\dot{7}74 & 193 & 548 & 387 & 09\dot{6} & \\
.\dot{1}93 & 548 & 387 & 096 & 77\dot{4} & \\
.\dot{5}48 & 387 & 096 & 774 & 19\dot{3} & \\
.\dot{3}87 & 096 & 774 & 193 & 54\dot{8} & \\
\hline
1.\dot{9}98 & 998 & 998 & 998 & 99\dot{8} & = .\dot{9} \times 2 \\
+ & + & + & + & & \\
1 & 1 & 1 & 1 & &
\end{array}$$

(ii) If q be the product of primes or powers of primes or be a power of a prime, then the summation of sections in some instances gives results similar to those obtained when q is a prime. For instance, the period of $\frac{1}{91} = \frac{1}{13 \times 7} = .\dot{0}1098\dot{9}$, where the first half and the second half of the period are complementary and $.01 + 09 + 89 = 99$. In other instances, however, variations occur, the general nature of which is to be understood from the inspection of a few examples.

For the periods of 21, viz., $.\dot{0}4761\dot{9}$ and $.\dot{9}5238\dot{0}$ ($\frac{2}{21} = .\dot{0}9523\dot{8}$), $.047 + 619 = 666$, $952 + 380 = 1332 = 4 \times 333$. The two sums together $= 2 \times 999$. But $.04 + 76 + 19 = 99$; $95 + 23 + 80 = 198 = 2 \times 99$. As in some cases in which 3 is a factor of q, the sections when added give $\frac{m}{3}(10^d - 1)$, so when 9 is a factor of q they sometimes give $\frac{m}{3}(10^d - 1)$. *E.g.*, for 117, $008 + 547 = 555$, but $00 + 85 + 47 = 132 = 4 \times 33$, $.99 + 14 + 52 = 165 = 5 \times 33$.

For the period of 49:

$$\begin{array}{ll}
 & \text{Corresponding numerators.} \\
.\dot{0}20408\ldots & 1 \\
.\dot{1}63265\ldots & 8 \\
.\dot{3}06122\ldots & 15 \\
.\dot{4}48979\ldots & 22 \\
.\dot{5}91836\ldots & 29 \\
.\dot{7}34693\ldots & 36 \\
.\dot{8}77551\ldots & 43 \\
\hline
3.\dot{1}42854 = .\dot{1}42857 \times 22 & 154 = 7 \times 22 \\
+ & \\
3\ldots = .14285\dot{7} \times 22 &
\end{array}$$

To generalize, if a period has a number of digits which is a multiple of the number (f) of digits in the period of a factor (r) of q, then sections of kf digits when added give $\frac{m}{r}(10^{kf}-1)$.

For the period of 221 ($=13\times 17$), $a=48$, $n=4$ (*i.e.*, two pairs of complementary periods). The period of $\frac{1}{13}$ is $.\dot{0}7692\dot{3}$.

Sections of 6 digits.	Corresponding remainders or numerators.
$.\dot{0}04524\ldots$	1st — $1 = 0\times 13+1$
$.\dot{8}86877\ldots$	7th — $196 = 15\times 13+1$
$.\dot{8}28054\ldots$	13th — $183 = 14\times 13+1$
$.\dot{2}98642\ldots$	19th — $66 = 5\times 13+1$
$.\dot{5}33936\ldots$	25th — $118 = 9\times 13+1$
$.\dot{6}51583\ldots$	31st — $144 = 11\times 13+1$
$.\dot{7}10407\ldots$	37th — $157 = 12\times 13+1$
$.\dot{2}39819\ldots$	43d — $53 = 4\times 13+1$
$4.\dot{1}53842 = 54\times .\dot{0}76923$	$918 = 54\times 17$
$+$	
$4\ldots\quad 54\times .\dot{0}7692\dot{3}$	

Similarly—

the 2d, 8th... 44th numerators are of the form $\pm m\,13+10$
and the 3d, 9th... 45th " " " " $\pm m\,13+100$
and the 4th, 10th...46th " " " " $\pm m\,13+116$
and so on.

The halves of the period of $\frac{1}{221}=\frac{7}{13}(10^{24}-1)$, and the quarters $=\frac{27}{13}(10^{12}-1)$, while the thirds $=\frac{12}{17}(10^{16}-1)$ and the numerators corresponding to the thirds $=12\times 13$. The sixths $=2(10^{8}-1)$. The other periods yield analogous results. Note that $m=0$ in the form of the first 6 numerators, and that the minus sign only occurs for some values of q. Analysis of this kind can be applied generally.

The following partial exhibition of the relations to each other and to 7 and 47 of the remainders of the period of $\frac{1}{7\times 47}=\frac{1}{329}$ may perhaps prove suggestive. There is one period of 6 digits to 7 and one period of 46 digits to 47, and two periods of 138 digits (the halves being complementary) to 329.

period of $\frac{1}{329}$ is .0̇0303.95136.77811.55015.19756.83890.57750.75987.84194.52887.53799.39209.72644.3768
99696.04863.22188.44984.80243.16109.42249.24012.15805.47112.46200.60790.27355.623i̇

1
10
100
13=7× 2− 1
130=7×20− 10
313=7×59−100
7×24+ 1=**169**=7×26− 13
7× 5+ 10= **45**=7×25−130
7× 3+100=**121**=7×62−313
7×30+ 13=**223**=7×56−169=7×32− 1
7×18+130=**256**=7×43− 45=7×38− 10
−7× 8+313=**257**=7×54−121=7×51−100
7×38+ 1= 7×14+169=**267**=7×70−223=7×40− 13
7× 4+ 10=−7× 1+ 45= **38**=7×42−256=7×24−130
−7× 7+100=−7×10+121= **51**=7×44−257=7×52−313
7×14+ 13=−7× 6+223=**181**=7×62−267=7×50−169=7×26− 1
7× 5+130=−7×13+256=**165**=7×29− 38=7×30− 45=7×27− 10
7×44+313=−7×36+257= **5**=7× 8− 51=7×18−121=7×15−100
7× 7+ 1=−7×17+169=−7×31+267= **50**=7×33−181=7×39−223=7× 9− 13
7×23+ 10= 7×18+ 45= 7×19+ 38=**171**=7×48−165=7×61−256=7×43−130
−7× 5+100=−7× 8+121= 7× 2+ 51= **65**=7×10− 5=7×46−257=7×54−313
7×44+ 13= 7×14+223= 7×20+181=**321**=7×53− 50=7×84−267=7×70−169=7×46− 1
7×17+130=−7× 1+256= 7×12+165=**249**=7×60−171=7×41− 38=7×42− 45=7×37− 10
−7×18+313=−7×10+257= 7×26+ 5=**187**=7×36− 65=7×34− 51=7×44−121=7×41−100=47× 4− 1
7×32+ 1= 7× 8+169=−7× 6+267= 7×25+ 50=**225**=7×78−321=7×58−181=7×64−223=7×34− 13=47× 5− 10

$7\times38+10= \; 7\times33+45= \; 7\times15+38= \; 7\times15+171=\mathbf{276}=7\times75-249=7\times63-165=7\times76-256=7\times58-130=47\times8-100$

$7\times4+100= \; 7\times1+121= \; 7\times11+51= \; 7\times9+65=\mathbf{128}=7\times45-187=7\times19-5=7\times55-257=7\times63-313=47\times3-13$

$7\times20+13= \; 7\times10+223= \; 7\times16+181=-7\times4+321=\mathbf{293}=7\times74-225=7\times49-50=7\times80-267=7\times66-169=47\times7-130$

$7\times42-1$

$7\times14+130= \; 7\times6+256= \; 7\times19+165= \; 7\times7+249=\mathbf{298}=7\times82-276=7\times67-171=7\times48-38=7\times49-45=7\times44-10=47\times13-313$

$-7\times42+313=-7\times34+257= \; 7\times2+5=-7\times24+187= \; \mathbf{19}=7\times21-128=7\times12-65=7\times10-51=7\times20-121=7\times17-100=47\times4-169$

$7\times27+1= \; 7\times3+169=-7\times11+267= \; 7\times20+50=-7\times5+225=\mathbf{190}=7\times69-293=7\times73-321=7\times53-181=7\times59-223=7\times29-13=47\times5-45$

$7\times35+10= \; 7\times30+45= \; 7\times31+38= \; 7\times12+171=-7\times3+276=\mathbf{255}=7\times79-298=7\times72-249=7\times60-165=7\times73-256=7\times55-130=47\times8-121$

$7\times21+100= \; 7\times18+121= \; 7\times28+51= \; 7\times25+65= \; 7\times17+128=\mathbf{247}=7\times38-19=7\times45-187=7\times36-5=7\times72-257=7\times80-313=47\times10-223$

$7\times22+13=-7\times8+223=-7\times2+181=-7\times22+321=-7\times18+293=\mathbf{167}$

$-7\times15+130=-7\times33+256=-7\times20+165=-7\times32+249=-7\times39+298=\mathbf{25}$

$-7\times9+313=-7\times1+257= \; 7\times35+5= \; 7\times9+187= \; 7\times33+19=\mathbf{250}$

$7\times23+1= \; 7\times2+169=-7\times10+267= \; 7\times21+50=-7\times4+225= \; 7\times1+190=\mathbf{197}$

325
289
258
277
138
7 multiples of $7+\dots$ **64**
311
149
174
95 (the 47th)
292
8 multiples of $7+\dots$ **288**
248
(the 70th) **328** (complementary)
(the 93d) **142**$=47\times10-328$ (the 70th) $=47\times7-187$ (the 24th) $=7\times65-313$

6 multiples of 7 $=47\times9-256$
$=47\times6-257$
$=47\times11-267$
$=44\times5-38$
$=47\times8-51$
$=47\times10-181$
7 multiples of 7 $=47\times9-165$
$=47\times6-5$
$=47\times4-50$
$=47\times5-171$
$=47\times8-65$
$=47\times10-321$
8 multiples of 7 $=47\times9-249$
$=47\times6-187$
$=47\times11-22$
$=47\times12-27$
$=47\times8-128$
$=47\times9-95=47\times7-1$

$47\times2+1=$
$47\times6+10=$
$47\times4+100=$
$47\times5+13=$

$7\times6+100=47\times3+1=$

§ 5. No explanation is here proposed of the following curious property of periods for which q is prime and $a = q - 1$, and is also divisible by 4; so that its universality is not deduced or assumed.

Let a section of m digits of a period be represented by G $(1, 2, 3 \ldots m)$, and G$(4 \ldots m)$ represent part of the section from the 4th digit to the mth or last digit, and G $(1 \ldots [m - 6])$ represent part of the section from the first to the $(m - 6\text{th})$ digit, and G $(x \ldots [m - y])$ represent a middle portion of the section from the xth digit to the $(m - y)$th digit. Let A $(1 \ldots \frac{q-1}{4})$, B $(1 \ldots \frac{q-1}{4})$, C $(1 \ldots \frac{q-1}{4})$, D $(1 \ldots \frac{q-1}{4})$, be the four sections of the period of $\frac{1}{q}$ in order. Arrange A $(1 \ldots \frac{q-1}{4})$ followed by C $(1 \ldots \frac{q-1}{4})$ over B $(1 \ldots \frac{q-1}{4})$, followed by D $(1 \ldots \frac{q-1}{4})$, making two ranks of digits, and add; then the sum E $(1 \ldots \frac{q-1}{2})$ will contain in order $\frac{q-1}{2}$ of the digits of the period. If, however, $q - 1$ be a multiple of 10, E $(1 \ldots \frac{q-1}{2})$ will contain only $\frac{q-1}{2} - 2$ of the said digits.

EXAMPLES.

For $\frac{1}{17}$ 05889411
23527647
———
29417058

For $\frac{1}{29}$ 03448274137931
58620689655172
———
62068963793103

For $\frac{1}{61}$ 016393442622950180327868852459
819672131147540983606557377049
———
836065573770491163934426229508

As this property is not shared by periods of q when n does not $= 1$, it cannot be altogether due to the halves of the periods being complementary. It appears to be due to the arrangement of all the periods of q under one cycle of digits.

§ 6. It is noteworthy that the only completely cyclic *number* is 142857.

For 142857 × 2 $= 285714$
" × 3 $= 428571$
" × 4 $= 571428$
" × 5 $= 714285$
" × 6 $= 857142$
" × $7m = m\,(10^6 - 1)$.

Hence, to multiply 142857 by any number, $7m + n$ (where $n = 1$ or 2 or 3 or 4 or 5 or 6), we have only to divide the multiplier by 7, thus finding m and n, prefix m to $142857 \times n$ and then subtract m.

Thus to find $(142857)^2$

$$7)\underline{142857}$$
$$20408 - 1$$
$$20408122449 = (142857)^2.$$

Also, $2915446064142857 - 2915446064 =$
$$2915443148696793 = (142857)^3$$

Stated Meeting, October 18, 1901.

Vice-President WISTAR in the Chair.

Present, 9 members.

Mr. Thomas Willing Balch, a newly elected member, was presented to the Chair, and took his seat in the Society.

The list of donations to the Library was laid on the table and thanks were ordered to be returned for them.

With reference to one of the donations, Dr. Hays called attention to a statement contained in Mr. William Eleroy Curtis's *True Thomas Jefferson*, just published, that this Society possessed Jefferson's "original draft" of the Declaration of Independence, with all the corrections. He thought it important that this statement should not remain uncorrected, as it might lead to considerable disappointment. The copy